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Maximality in finite-valued Lukasiewicz logics
defined by order filters

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1 Preliminaries and first results

In this talk we consider the logics \( L_n^i \), obtained from the \((n+1)\)-valued Lukasiewicz logics \( L_{n+1} \) by taking the order filter generated by \( i/n \) as the set of designated elements. The \((n+1)\)-valued Lukasiewicz logic can be semantically defined as the matrix logic

\[
L_{n+1} = \langle LV_{n+1}, \{1\} \rangle,
\]

where \( LV_{n+1} = (LV_{n+1}, \neg, \to) \) with \( LV_{n+1} = \{0, \frac{1}{n}, \ldots, \frac{n+1}{n}, 1\} \), and the operations are defined as follows: for every \( x, y \in LV_{n+1} \), \( \neg x = 1 - x \) and \( x \to y = \min\{1, 1 - x + y\} \).

Observe that \( L_2 \) is the usual presentation of classical propositional logic CPL as a matrix logic over the two-element Boolean algebra \( B_2 \) with domain \( \{0, 1\} \) and signature \( \{\neg, \to\} \). The logics \( L_n \) can also be presented as Hilbert calculi that are axiomatic extensions of the infinite-valued Lukasiewicz logic \( L_\infty \).

The following operations can be defined in every algebra \( LV_{n+1} \): \( x \ominus y = \neg (x \to \neg y) = \max\{0, x + y - 1\} \) and \( x \oplus y = \neg x \to y = \min\{1, x + y\} \). For every \( n > 1 \), \( x^n = x \ominus \cdots \ominus x \) (\( n \)-times) and \( nx = x \oplus \cdots \oplus x \) (\( n \)-times).

For \( 1 \leq i \leq n \) let \( F_{i/n} = \{x \in LV_{n+1} : x \geq i/n\} = \{\frac{i}{n}, \ldots, \frac{n+1}{n}, 1\} \) be the order filter generated by \( i/n \), and let

\[
L_n^i = \langle LV_{n+1}, F_{i/n} \rangle
\]

be the corresponding matrix logic. From now on, the consequence relation of \( L_n^i \) is denoted by \( \models_{L_n^i} \). Observe that \( L_{n+1} = L_n^1 \) for every \( n \). In particular, \( CPL \) is \( L_2^1 \) (that is, \( L_2 \)). If \( 1 \leq i, m \leq n \), we can also consider the following matrix logic: \( L_n^{i/m} = \langle LV_{m+1}, F_{i/m} \cap LV_{m+1} \rangle \).

The logic \( L_3^1 = \langle LV_3, \{1, 1/2\} \rangle \) was already known as the 3-valued paraconsistent logic \( J_3 \), introduced by da Costa and D’Ottaviano see [4] in order to obtain an example of a paraconsistent logic maximal w.r.t. CPL.

**Definition 1.** Let \( L_1 \) and \( L_2 \) be two standard propositional logics defined over the same signature \( \Theta \) such that \( L_1 \) is a proper sublogic of \( L_2 \). Then, \( L_1 \) is **maximal w.r.t.** \( L_2 \) if, for every formula \( \phi \) over \( \Theta \), if \( \vdash_{L_2} \phi \) but \( \not\vdash_{L_1} \phi \), then the logic \( L_1^+ \) obtained from \( L_1 \) by adding \( \phi \) as a theorem, coincides with \( L_2 \).

In order to study maximality among finite-valued Lukasiewicz logics defined by order filters we obtain the following sufficient condition:
Theorem 1. Let $L_1 = \langle A_1, F_1 \rangle$ and $L_2 = \langle A_2, F_2 \rangle$ be two distinct finite matrix logics over a same signature $\Theta$ such that $A_2$ is a subalgebra of $A_1$ and $F_2 = F_1 \cap A_2$. Assume the following:

1. $A_1 = \{0, 1, a_1, \ldots, a_k, a_{k+1}, \ldots, a_n\}$ and $A_2 = \{0, 1, a_1, \ldots, a_k\}$ are finite such that $0 \notin F_1$, $1 \in F_2$ and $\{0, 1\}$ is a subalgebra of $A_2$.

2. There are formulas $\top(p)$ and $\bot(p)$ in $\mathcal{L}(\Theta)$ depending at most on one variable $p$ such that $e(\top(p)) = 1$ and $e(\bot(p)) = 0$, for every evaluation $e$ for $L_1$.

3. For every $k + 1 \leq i \leq n$ and $1 \leq j \leq n$ (with $i \neq j$) there exists a formula $\alpha_i^j(p)$ in $\mathcal{L}(\Theta)$ depending at most on one variable $p$ such that, for every evaluation $e$, $e(\alpha_i^j(p)) = a_j$ if $e(p) = a_i$.

Then, $L_1$ is maximal w.r.t. $L_2$.

We use this result to prove that

Theorem 2. Let $1 \leq i, m \leq n$. Then $L_i^m$ is maximal w.r.t. $L_i^{0m}$ if the following condition holds: there is some prime number $p$ and $k \geq 1$ such that $n = p^k$, and $m = p^{k-1}$.

Corollary 1. Let $1 \leq i \leq p$. For every prime number $p$, $L_i^p$ is maximal w.r.t. CPL.

Notice that the above corollary generalizes the well known result: $L_{p+1}$ is maximal w.r.t. CPL for every prime number $p$.

Definition 2. Let $L_1$ and $L_2$ be two standard propositional logics defined over the same signature $\Theta$ such that $L_1$ is a proper sublogic of $L_2$. Then, $L_1$ is strongly maximal w.r.t. $L_2$ if, for every finitary rule $\varphi_1, \ldots, \varphi_n / \psi$ over $\Theta$, if $\varphi_1, \ldots, \varphi_n \vdash_{L_2} \psi$ but $\varphi_1, \ldots, \varphi_n \not\vdash_{L_1} \psi$, then the logic $L_1^i$ obtained from $L_1$ by adding $\varphi_1, \ldots, \varphi_n / \psi$ as structural rule, coincides with $L_2$.

Let $i$ be a strictly positive integer, the $i$-explosion rule is the rule $(\text{exp}_i) \frac{i(\varphi \land \lnot \varphi)}{\bot}$.

Lemma 1. For every $1 \leq i \leq n$, the rule $(\text{exp}_i)$ is not valid in $L_i^i$.

Corollary 2. Let $1 \leq i \leq p$. For every prime number $p$, $L_i^p$ is not strongly maximal w.r.t. CPL.

2 Equivalent systems

Blok and Pigozzi introduce in [3] the notion of equivalent deductive systems in the following sense: Two propositional deductive systems $S_1$ and $S_2$ in the same language $\mathcal{L}$ are equivalent if there are two translations $\tau_1, \tau_2$ (finite subsets of $\mathcal{L}$-propositional formulas in one variable) such that:

- $\Gamma \vdash_{S_1} \varphi$ iff $\tau_1(\Gamma) \vdash_{S_2} \tau_1(\varphi)$,
- $\Delta \vdash_{S_2} \psi$ iff $\tau_2(\Delta) \vdash_{S_1} \tau_2(\psi)$,
- $\varphi \vdash_{S_1} \tau_1(\varphi)$,
- $\psi \vdash_{S_2} \tau_1(\tau_2(\psi))$.

Theorem 3. For every $n \geq 2$ and every $1 \leq i \leq n$, $L_i^n$ and $L^{n+1}$ are equivalent deductive systems.
From the equivalence among $L^i_n$ and $L_{n+1}$, we can obtain, by translating the axiomatization of the finite valued Lukasiewicz logic $L_{n+1}$, a calculus sound and complete with respect $L^i_n$ that we denote by $H^i_n$.

Since $L^i_n$ is algebraizable and the class $MV$ of all MV-algebras is its equivalent quasivariety semantics, finitary extensions of $L^i_n$ are in 1 to 1 correspondence with quasivarieties of MV-algebras. Actually, there is a dual isomorphism from the lattice of all finitary extensions of $L^i_n$ and the lattice of all quasivarieties of $MV$. Moreover, if we restrict this correspondence to varieties of $MV$ we get the dual isomorphism from the lattice of all varieties of $MV$ and the lattice of all axiomatic extensions of $L^i_n$. Since $L_{n+1} = L^n_n$ is an axiomatic extension of $L^i_n$, $L_{n+1}$ is an algebraizable logic with the class $MV_n = Q(LV_{n+1})$, the quasivariety generated by $LV_{n+1}$, as its equivalent variety semantics. It follows from the previous theorem that $L^i_n$, for every $1 \leq i \leq n$, is also algebraizable with the same class of $MV_n$-algebras as its equivalent variety semantics. Thus, the lattices of all finitary extensions of $L^i_n$ are isomorphic, and in fact, dually isomorphic to the lattice of all subquasivarieties of $MV_n$, for all $0 < i < n$.

Therefore maximality conditions in the lattice of finitary (axiomatic) extensions correspond to minimality conditions in the lattice of subquasivarieties (subvarieties). Thus, given two finitary extensions $L_1$ and $L_2$ of a given logic $L^i_n$, where $K_{L_1}$ and $K_{L_2}$ are its associated $MV$-quasivarieties, $L_1$ is strongly maximal with respect $L_2$ if $K_{L_1}$ is a minimal subquasivariety of $MV_n$ among those $MV_n$-quasivarieties properly containing $K_{L_2}$. Moreover, if $L_1$ and $L_2$ are axiomatic extensions of $L^i_n$, then $K_{L_1}$ and $K_{L_2}$ are indeed $MV_n$-varieties. In that case, $L_1$ is maximal with respect $L_2$ if $K_{L_1}$ is a minimal subvariety of $MV_n$ among those $MV_n$-varieties properly containing $K_{L_2}$.

The lattice of all axiomatic extensions $L^i_n$ is fully described also by Komori in [7], thus from the equivalence of Theorem 3, we can obtain the following maximality conditions for all axiomatic extensions of $L^i_n$.

**Theorem 4.** Let $0 < i, m \leq n$ be natural numbers such that $m \leq n$. If $L$ is an axiomatic extension of $L^i_n$, then $L$ is maximal with respect $L^i_n$ if $L = L^i_n \cap L^i_m$ for some prime number $p$ with $p | m$ and a natural $k \geq 0$ such that $p^k | m$ and $p^{k+1} \notin m$.

As a corollary we obtain that the sufficient condition of Theorem 2 is also necessary.

**Corollary 3.** Let $1 \leq i, m \leq n$. Then $L^i_n$ is maximal w.r.t. $L^i_m$ if and only if there is only if some prime number $p$ and $k \geq 1$ such that $n = p^k$, and $m = p^{k+1}$.

To obtain results on strong maximality we need to study finitary extensions of $L^i_n$. The task of fully describing the lattice of all finitary extensions of $L^i_n$, isomorphic to the lattice of all quasivarieties of $MV$, turns to be an heroic task since the class of all MV-algebras is $Q$-universal [1]. For the finite valued case it is much simpler, since $MV_n$ is a locally finite discriminator variety. Any locally finite quasivariety is generated by its critical algebras [5]. Critical MV-algebras were fully described in [6] and using this description we can obtain some results on strong maximality.

First we need to introduce the following matrix logics: For every $1 \leq i, m \leq n$,

$$L^i_n = (LV_{n+1} \times LV_2, F_{i/n}) \quad L^i_m = (LV_{m+1} \times LV_2, (F_{i/n} \cap LV_{m+1}) \times \{1\})$$

**Theorem 5.** Let $0 < i \leq n$ be natural numbers, let $p$ be a prime number and let $r = \max\{j \in N : p^j | n\}$. Then we have: For every $j$ such that $(i - 1)p < j \leq ip$, $L^i_n \cap L^{i,rp+1}_{p+1}$ is strongly maximal with respect to $L^i_n$. Moreover, every finitary extension of some $L^i_n$ is strongly maximal with respect $L^i_n$ if it is one of the preceding types.
Theorem 6. Let \( p \) be a prime number. Then, for every \( j \) such that \( 0 < j \leq p \):

- \( \bar{L}_j \) is strongly maximal with respect to CPL and it is axiomatized by \( H_j^p \) plus the \( j \)-explosion rule \((exp_j)\) \( j(\varphi \land \neg \varphi)/\bot \).
- \( \bar{L}_j \) is strongly maximal w.r.t. \( \bar{L}_j^p \).

3 Ideal paraconsistent logics


Definition 3. Let \( L \) be a propositional logic defined over a signature \( \Theta \) (with consequence relation \( \vdash_L \)) containing at least a unary connective \( \neg \) and a binary connective \( \rightarrow \) such that:

(i) \( L \) is paraconsistent w.r.t. \( \neg \), i.e. there are formulas \( \varphi, \psi \in L(\Theta) \) such that \( \varphi, \neg \varphi \not\vdash_L \psi \); and \( \rightarrow \) is a deductive implication, i.e. \( \Gamma \cup \{ \varphi \} \vdash_L \psi \iff \Gamma \vdash_L \varphi \rightarrow \psi \).

(ii) There is a presentation of CPL as a matrix logic \( L^i \equiv (A, \{1\}) \) over the signature \( \Theta \) such that the domain of \( A \) is \( \{0, 1\} \), and \( \neg \) and \( \rightarrow \) are interpreted as the usual 2-valued negation and implication of CPL, respectively, such that \( L \) is a sublogic of CPL.

Then, \( L \) is said to be an ideal paraconsistent logic if it is maximal w.r.t. CPL, and every proper extension of \( L \) over \( \Theta \) is not \( \neg \)-paraconsistent.

Lemma 2. Let \( 0 < i \leq n \). \( L_n^i \) is paraconsistent w.r.t. \( \neg \) iff \( \frac{i}{n} \leq \frac{1}{2} \).

Since for every \( 0 < i \leq n \), there is a term definable implication \( \Rightarrow_n^i \) which is deductive implication next result follows from Theorem 6.

Theorem 7. Let \( p \) be a prime number, and let \( 1 < i < p \) such that \( i/p \leq 1/2 \). Then, \( L_i^p \) is a \((p+1)\)-valued ideal paraconsistent logic.\(^1\)

References


\(^1\)Strictly speaking, in this claim we implicitly assume that the signature of \( L_i^p \) has been changed by adding the definable implication \( \Rightarrow_n^i \) as a primitive connective.